AP Calculus
Stuff You Must Know

Trig Stuff
Identities

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

$\sin^2 x = \dfrac{1-\cos 2x}{2}$

$\cos^2 x = \dfrac{1+\cos 2x}{2}$

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\sec x = \dfrac{1}{\cos x}$

$\csc x = \dfrac{1}{\sin x}$

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$\tan(-x) = -\tan x$

$\cot(-x) = -\cot x$

$\sec(-x) = \sec x$

$\csc(-x) = -\csc x$

Trig Values

θ	sin θ	cos θ	tan θ
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	∞

Algebra Stuff

Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Point-slope form: $y - y_0 = m(x - x_0)$

Standard form: $Ax + By = C$

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Differential Calculus
Derivative Formulas and Rules

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(\sin x\right) = \cos x$$

$$\frac{d}{dx}\left(\cos x\right) = -\sin x$$

$$\frac{d}{dx}\left(\tan x\right) = \sec^2 x$$

$$\frac{d}{dx}\left(\cot x\right) = -\csc^2 x$$

$$\frac{d}{dx}\left(\sec x\right) = \sec x \tan x$$

$$\frac{d}{dx}\left(\csc x\right) = -\csc x \cot x$$

$$\frac{d}{dx}\left(e^x\right) = e^x$$

$$\frac{d}{dx}\left(a^x\right) = a^x \ln a$$

$$\frac{d}{dx}\left(\ln x\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\log_b x\right) = \frac{1}{x \ln b}$$

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(\cot^{-1} x\right) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(\csc^{-1} x\right) = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(uv\right) = uv' + vu'$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))g'(x)$$

Uses of the first and second derivative

Curve Sketching

- $y = f(x)$ must be a continuous function on the given interval.
- To find a critical value, set $f'(x) = 0$ or undefined.
- Use a labeled first-derivative chart to determine if the function has a relative max or min. Make sure you write sentences explaining why.
- Alternately, relative max. If $f''(x_0) = -$ you can use the Second Derivative Test. If $f''(x_0) = +$, then x_0 is the x-coordinate of the relative maximum. If $f''(x_0) = -$, then x_0 is the x-coordinate of the relative minimum.
- To find points of inflection, set $f''(x) = 0$ or undefined. Use a labeled second-derivative sign chart to show that the sign of $f''(x)$ changes at that point.

Three Important Theorems

Intermediate Value Theorem

If a function, $f(x)$ is continuous on a closed interval [a, b], and *y* is some value between *f(a)* and *f(b)*, then there exists at least one number *x* = *c* in the open interval (a, b) where $f(c) = y$.

In simple words, the function must pass through every y-value between *f(a)* and *f(b).*

Mean Value Theorem for Derivatives

If the function, $f(x)$ is continuous on the closed interval [a, b] AND $f(x)$ is differentiable on the open interval (a, b), then there exists at least one number *x* = *c* in the open interval (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

In simple words, there is at least one point in the interval (a, b) where the slope of the tangent line to a smooth curve is parallel to the secant line drawn through the endpoints of the interval.

Rolle's Theorem

If the function, $f(x)$ is continuous on the closed interval [a, b] AND $f(x)$ is differentiable on the open interval (a, b), AND *f(a)* = *f(b)*, then there exists at least one number *x* = *c* in the open interval (a, b) such that $f'(c) = 0$.

In simple words, if the endpoints of the interval of a differentiable function have the same y-coordinates, then there is at least one point in the interval (a, b) where the slope of the tangent line is equal to zero.
This is really a special case of the Mean Value Theorem.

Integral Formulas

$\int x^n dx = \frac{x^{n+1}}{n+1} + c;\ n \neq -1$

$\int \frac{1}{x} dx = \ln|x| + c$

$\int e^x dx = e^x + c$

$\int a^x dx = \frac{a^x}{\ln a} + c$

$\int \sin x\, dx = -\cos x + c$

$\int \cos x\, dx \sin x + c$

$\int \tan x\, dx = -\ln|\cos x| + c$

$\int \cot x\, dx = \ln|\sin x| + c$

$\int \sec x\, dx = \ln|\sec x + \tan x| + c$

$\int \csc x\, dx = -\ln|\csc x + \cot x| + c$

$\int \sec^2 x\, dx = \tan x + c$

$\int \csc^2 x\, dx = -\cot x + c$

$\int \sec x \tan x\, dx = \sec x + c$

$\int \csc x \cot x\, dx - \csc x + c$

$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + c$

$\int \frac{1}{1+x^2} dx = \arctan x + c$

$\int \frac{1}{x\sqrt{x^2-1}} dx = arc \sec x + c$

$\int \ln x\, dx = x \ln x - x + c$

$\int u\, dv = uv - \int v\, du + c$

(integration by parts)

Fundamental Theorem of Calculus-- Part 1

$\int_a^b f(x)\, dx = F(b) - F(a)$

Where $F'(x) = f(x)$

Fundamental Theorem of Calculus-- Part 2

$$\frac{d}{dx}\left(\int_a^x f(t)\, dt\right) = f(x)$$

Average Value Theorem

If the function $f(x)$ is continuous on the closed interval [a, b], there exists some number *c* such that

$$Avg\,Value = f(c) = \frac{1}{b-a}\int_a^b f(x)\, dx$$

Trapezoidal Rule

$$\int_a^b f(x)\, dx = \frac{1}{2}\left(\frac{b-a}{n}\right) \times$$
$$\left[f(x_0) + 2f(x_1) + \ldots + 2f(x_{n-1}) + f(x_n)\right]$$

Volume of a Solid of Revolution

Disk Method

$$V = \pi \int_a^b \left((OR)^2 - (IR)^2\right) dx \ \ or\, dy$$

Volume of a Solid of Known Cross-Section

$$V = \int_a^b Area(x)\, dx$$

Particle Motion Formulas

velocity $= \frac{d}{dt}(\text{position})$	accleration $= \frac{d}{dt}(\text{velocity})$	displacement $= \int_{t_1}^{t_2} v(t)\,dt$
total distance $= \int_{t_1}^{t_2} \lvert v(t)\rvert\,dt$	average velocity $= \dfrac{\text{final position - initial position}}{\text{total time}}$	arc length $= \int_a^b \sqrt{1+(f'(x))^2}\,dx$
speed $= \lvert\text{velocity}\rvert = \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$	velocity vector $= \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$	acceleration vector $= \left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right\rangle$

Parametric Equations

Slope = $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$

Second Derivative in Parametric Form

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

Polar Curves

Slope

$x = r\cos\theta$

$y = r\sin\theta$

Slope = $\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

Area inside a polar curve $= \frac{1}{2}\int_{\theta_1}^{\theta_2} [r(\theta)]^2\,d\theta$

where θ_1 and θ_2 are the first two times that $r = 0$.

L'Hôpital's Rule

If $\dfrac{f(x)}{g(x)} = \dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$, then $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)} = \lim\limits_{x\to a}\dfrac{f'(x)}{g'(x)}$

Euler's Method

Given that $\dfrac{dy}{dx} = f(x,y)$ and that the solution passes through (x_0, y_0), then

$$x_{new} = x_{old} + \Delta x$$
$$y_{new} = y_{old} + \left.\frac{dy}{dx}\right|_{(x_{old},\, y_{old})} \times \Delta x$$

Series Stuff

Taylor Series

If a function *f* is "smooth" at $x = a$, then *f* can be approximated by the n^{th} degree polynomial

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Maclaurin Series

A Taylor series centered about $x = 0$ is called a Maclaurin Series.

Familiar Maclaurin Series you must know

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots + x^n + \ldots$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots + (-1)^n x^n + \ldots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots + (-1)^{n+1} x^n + \ldots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + (-1)^n \frac{x^{2n}}{(2n)!} + \ldots$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} + \ldots + (-1)^{2n+1} \frac{x^{2n+1}}{2n+1}$$

Convergence Tests

Ratio Test: The series $\sum_{k=0}^{\infty} a_k$ converges if $\lim\limits_{k\to\infty}\left|\dfrac{a_{k+1}}{a_k}\right| < 1$

nth Term Test: If $\lim\limits_{n\to\infty} a_n \neq 0$, then the series $\sum_{k=0}^{\infty} a_k$ diverges.

Direct Comparison Test: If $a_n \le c_n$ and $\sum_{k=1}^{\infty} c_k$ converges, then $\sum_{k=1}^{\infty} a_k$ will also converge.